Applicable Analysis and Discrete Mathematics, **1** (2007), 360–370.
Available electronically at **http://pefmath.etf.bg.ac.yu**

ON TYPES, FORM AND SUPREMUM OF THE SOLUTIONS OF THE LINEAR DIFFERENTIAL EQUATION OF THE SECOND ORDER WITH ENTIRE COEFFICIENTS

Dragan Dimitrovski, Miloje Rajović, Rade Stojiljković

Some new, basically combined classical procedures for qualitative analysis of the equation

$$y'' + a(x)\, y' + b(x)\, y = 0,$$

if $a(x)$ and $b(x)$ are continuously differentiable coefficients, are given in this paper, in the sense of general form of the solution, integral equations for the forms of the solution and estimation of the supremum of the solution.

The aim of this paper is to show that the variety of solutions of the very important differential equation

$$y'' + a(x)\, y' + b(x)\, y = 0, \tag{1}$$

if the coefficients are entire on the semi-infinite line $[0, +\infty)$, is very small and that the solutions are actually divided into two distinct classes: the first class include monotonous solutions of exponential type, the second class comprise oscillating functions, taken into account generally, very similar to ordinary $\sin x$ and $\cos x$.

The huge variety of the solutions of equation (1), well-known from the Theory of Special Functions, is in most cases a consequence of singularities of the coefficients $a(x)$ and $b(x)$ as well as the leading coefficient, $A(x)$, in the formula (1.A) at the end of the paper.

The famous thought of the great analyst Gurs: "Give me singularities and I will tell you about the function" is confirmed by this.

There are two basic classes of solutions of equation (1) with entire coefficients:

2000 Mathematics Subject Classification. 34A30, 34C20.
Keywords and Phrases. Second order differential equation with entire coefficients.